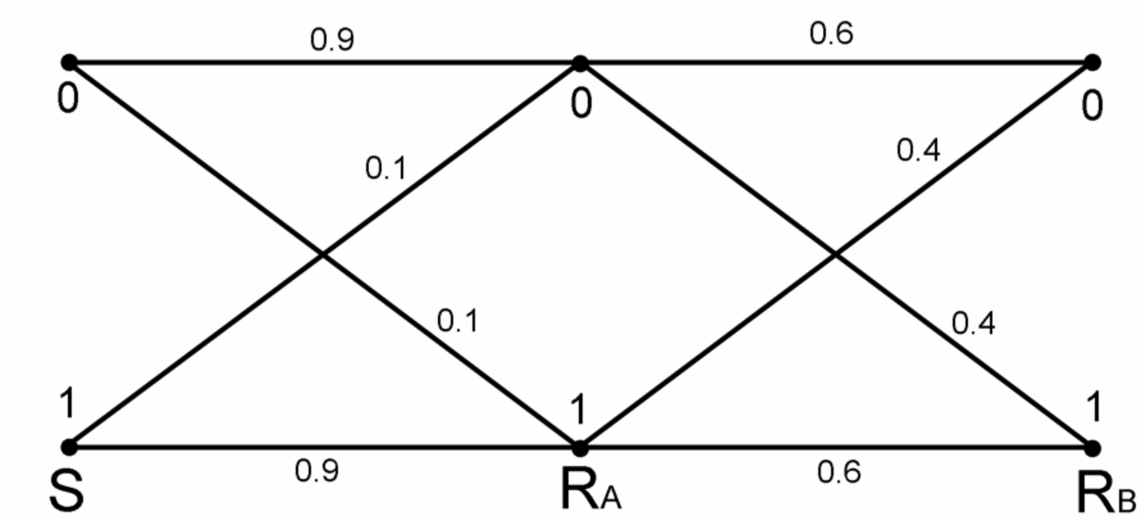
ECE139 – SPRING 2005 – Friday Discussion – 22 April 2005

You are given the following communications model. S is the symbol that is sent. P[S=1] = P[S=0] = 0.5. RA is the symbol received at station A. RB is the symbol received at station B. Station A transmits exactly what it receives. This is two binary channels in series, each with different probabilities for successful and erroneous transmissions, as given in the figure. For example, P[RB=0 | RA = 1] = 0.4 meaning there is a 40% chance of an error when station A sends a '1.'



1.. Find P[RA = 1]

Here, we must use Total Probability:
P[RA=1] = P[RA=1 | S=0]P[S=0] + P[RA=1 | S=1]P[S=1]
And we can just plug in:
P[RA=1] = 0.1*0.5 + 0.9*0.5

P[RA=1] = 0.5

2.. Find P[RB=1 | S=0]

First, we use Bayes' Rule:
P[RB=1 | S=0] = P[RB = 1, S=0] / P[S=0]

Now, we know that P(A) = P(A, B) + P(A, B^c), so
P[RB=1 | S=0] = P[RB = 1, RA=0, S=0] / P[S=0] + P[RB = 1, RA=1, S=0] / P[S=0]

Use Bayes again:
= P[RB = 1 | RA=0, S=0]P[RA=0, S=0] / P[S=0] + P[RB = 1 | RA=1, S=0]P[RA=1, S=0] / P[S=0]
= P[RB = 1 | RA=0, S=0]P[RA=0 | S=0]P[S=0] / P[S=0] +
P[RB = 1 | RA=1, S=0]P[RA=1 | S=0]P[S=0] / P[S=0]
We can cancel out the P[S=0] from both terms and we are left with
P[RB=1 | S=0] = P[RB = 1 | RA=0, S=0]P[RA=0 | S=0] + P[RB = 1 | RA=1, S=0]P[RA=1 | S=0]
Now, we know that RB depends only on RA, so P[RB = 1 | RA=0, S=0] = P[RB = 1 | RA=0] and we are left with:
P[RB=1 | S=0] = P[RB = 1 | RA=0]P[RA=0 | S=0] + P[RB = 1 | RA=1]P[RA=1 | S=0]
All of whose terms we know from the figure:
P[RB=1 | S=0] = 0.4 * 0.9 + 0.6 * 0.1

P[RB=1 | S=0] = 0.42

3.. Find P[S=0 | RB=1]

Once again we apply Bayes' Rule:
P[S=0 | RB=1] = P[RB=1 | S=0]P[S=0]/P[RB=1]

We know P[RB=1 | S=0] from #2 and P[S=0] is given.

We turn our attention to P[RB=1]:
P[RB=1] = P[RB=1 | RA=0]P[RA=0] + P[RB=1 | RA=1]P[RA=1]

The we expand P[RA=0] and P[RA=1]
= P[RB=1 | RA=0]P[RA=0 | S=0]P[S=0] + P[RB=1 | RA=0]P[RA=0 | S=1]P[S=1]
+ P[RB=1 | RA=1]P[RA=1 | S=0]P[S=0] + P[RB=1 | RA=1]P[RA=1 | S=1]P[S=1]

Which, we can see, represents the 4 possible paths to RB=1 in the figure. We can now proceed as in #2, but I'll skip the details here.
P[RB=1] = 0.5*(0.1*0.4 + 0.9*0.6 + 0.9*0.4 + 0.1*0.6)
= 0.5

P[S=0 | RB=1] = 0.5 * 0.42 / 0.5 = 0.42