

ECE 139
SPRING 2005
FRIDAY 10A-12P DISCUSSION
QUIZ #6
6 MAY 2005

NAME:

CHOOSE ONE OF THE TWO PROBLEMS.

#1 . You have a continuous random variable X with pdf

$$f_X(x) = \begin{cases} kx & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- a. find k
- b. find $E[X]$, the expected value of X
- c. find $\text{VAR}[X]$, the variance of X
- d. find $E[X^2]$ (you can do this without integrating)

#2 . You have a Gaussian random variable X with mean 0 and variance 1. ($X \sim N(0,1)$). You define the random variable Y as $Y = u(X)$ where $u(X)$ is the unit step function. What is the pmf of Y (you can write it out or plot it)? What kind of random variable is Y ?

FRIDAY 10A-12P QUIZ #6

① $f_X(x) = \begin{cases} kx & 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$

② $k \int_0^1 x dx = 1$, $k \int_0^1 x dx = \frac{k}{2} x^2 \Big|_0^1 = \frac{k}{2} = 0 \Rightarrow \boxed{k=2}$

③ $E[X] = \int_0^1 x f_X(x) dx = 2 \int_0^1 x^2 dx = \frac{2}{3} x^3 \Big|_0^1 = \boxed{\frac{2}{3}}$

④ $\text{VAR}[X] = E[(X - \mu_X)^2] = E[X^2] - E^2[X]$

so $E[X^2] = 2 \int_0^1 x^3 dx = \frac{1}{2} x^4 \Big|_0^1 = \frac{1}{2}$

and $\text{VAR}[X] = \frac{1}{2} - \frac{4}{9} = \boxed{\frac{1}{18}}$

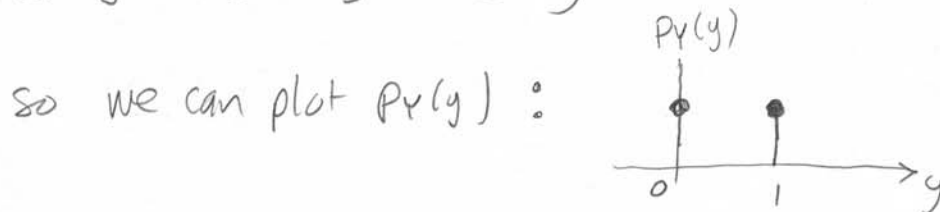
⑤ from ④, we have $\boxed{E[X^2] = \frac{1}{2}}$

② $Y = u(X)$ can only take on two values: 0 and 1

$Y=0$ if $X < 0$, $Y=1$ if $X \geq 0$

$P[Y=0] = P[X < 0] = \frac{1}{2}$
 $P[Y=1] = P[X \geq 0] = \frac{1}{2}$

} we know this from the symmetry of the Gaussian pdf.



And we see that this is a Bernoulli R.V.