

ECE 139  
SPRING 2005  
FRIDAY 10A-12P DISCUSSION  
QUIZ #7  
13 MAY 2005

NAME:

1.  $X$  is a Uniform random variable over the interval  $[-\pi/2, \pi/2]$  (i.e.  $X \sim \text{UNIF}[-\pi/2, \pi/2]$ ). We set the random variable  $Y = \sin(X)$ .

- a. What is the pdf of  $X$ ?
- b. What is  $E[Y]$ , the expected value of  $Y$ ?
- c. What is  $f_Y(y)$ , the pdf of  $Y$ ?

FRIDAY 10A-12P

QUIZ # 7.

1.  $X \sim \text{UNIF}(-\frac{\pi}{2}, \frac{\pi}{2})$ ,  $Y = \sin(X)$

$$\textcircled{a} f_X(x) = \frac{1}{b-a} = \frac{1}{\frac{\pi}{2} + \frac{\pi}{2}} = \begin{cases} \frac{1}{\pi} & -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\textcircled{b} E[Y] = E[\sin(X)] = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \sin x \, dx = \frac{-1}{\pi} \cos x \Big|_{-\pi/2}^{\pi/2} = 0$$

$$\textcircled{c} P[Y \leq y] = P[\sin x \leq y] = P[x \leq \sin^{-1}(y)]$$

$$= \frac{1}{\pi} \int_{-\pi/2}^{\sin^{-1}y} dx = \frac{1}{\pi} \left( \sin^{-1}y + \frac{\pi}{2} \right)$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} \frac{1}{\pi} \left( \sin^{-1}y + \frac{\pi}{2} \right) = \frac{1}{\pi} \frac{1}{\sqrt{1-y^2}} \quad -1 \leq y \leq 1$$

-OR-

$$f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \Big|_{x=x_k} = \frac{f_X(\sin^{-1}y)}{\cos(\sin^{-1}y)} = \frac{1}{\pi} \frac{1}{\sqrt{1-y^2}} \quad -1 \leq y \leq 1$$

NOTE:  $Y = \sin X$  for  $-\frac{\pi}{2} \leq X \leq \frac{\pi}{2}$

