Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

Modeling and Observer Design for an Array of Electrostatically Actuated Microcantilevers

Mariateresa Napoli[1] Bassam Bamieh[2]

Department of Mechanical Engineering,
University of California,
Santa Barbara, CA 93106,
U.S.A.

Abstract

In this paper we present a mathematical model for the dynamics of an array of capacitively actuated microcantilevers. We propose a system where the current measured at each cantilever is used as the sensing signal of the cantilever state through an observer. We show that such an array is a spatially invariant system with distributed control and sensing. For the common case of periodically excited cantilevers, we show that the underlying dynamics are those of a periodic system described by a Mathieu equation. We exploit the spatial invariance of the problem to design an optimal distributed observer, where the temporal periodicity is handled using the lifting technique.

1 Introduction

Over the past years, scanning-probe microscopes have proven to be extremely versatile instruments, for applications that include, but are not limited to, high resolution (atomic scale) surface imaging, high density (Gb/cm^2) data storage and retrieval, and optical lithography for advanced device processing.

The prospect for making miniaturized devices, using batch processing techniques, has brought the promise of obtaining these very high levels of performance at a limited cost. However, in order to achieve the anticipated results, an increase in throughput is required.

Research has evolved along two main lines: the integration of sensors and actuators, and the use of array architectures of the probes.

The most common solutions for integrated detection schemes make use of the piezoresistive [4, 6], piezoelectric [7, 8, 9], thermal expansion [10] or capacitive effects [2, 11]. A major advantage of capacitive detection, is the fact that it offers both electrostatic actuation as well as integrated detection. In particular, the novelty of our sensing scheme lies in that detection is *indirect*, meaning that a state observer provides an estimate of the cantilever displacement, based on the measurement of the current generated. From the point of view of implementation, this is a considerable advantage, since it involves the use of simpler circuitry.

In this paper, we propose a model for this electrostatically actuated microcantilever. More precisely, the microcantilever constitutes the movable plate of a capacitor and its displacement is controlled by the voltage applied across the plates. We show that its dynamics are governed by a special second order linear periodic differential equation, called the Mathieu equation. After formulating the optimal observer problem for this periodic system, we demonstrate how its solution can be approximated to any prescribed degree of accuracy by solving an almost equivalent problem for a certain discrete LTI system, obtained by lifting [3] and fast-sampling the original periodic model. We use this optimal design as an analysis tool to select the frequency of excitation that corresponds to the best achievable performance. This procedure, given the set of physical parameters of the system provides our model with the best combination of parameters to make it more easily observable. Finally, we design for this system a suboptimal reduced order observer, whose parameters are tuned to match the optimal performance index as close as possible.

As a second step we consider the connection of several cantilevers in an array architecture, where each cantilever is independently actuated and sensed. Currently, microcantilever arrays are designed with large spacing between the individual elements. This essentially decouples the dynamics of the individual cantilevers, that can be considered to behave as isolated units. The drawback of this configuration is, of course, a decrease in the potential throughput of the device. In this paper we want to model a tightly packed array of microcantilevers, explicitly incorporating their dynamical coupling into the model equations. We show that this system is an example of a spatially invariant system with distributed control and sensing. By exploiting the spatial invariance of the problem, it is possible to design a distributed optimal observer that "electronically" decouples the system, using the methodology of [12, 13, 14] and the results derived for the single cantilever case. Research along this line is currently being pursued.

The paper is organized as follows: In Section 2 we develop the mathematical model of the electrostatically actuated cantilever. In Section 3 we formulate the optimal observer

[1]e-mail: napoli@engineering.ucsb.edu
[2]e-mail: bamieh@engineering.ucsb.edu.

problem and show how to obtain an approximate solution via lifting and fast-sampling. In Section 4 we use this result as a guideline for the design of a suboptimal reduced order observer. In Section 4 we propose a model for the array architecture, and finally we present our conclusions in Section 5.

2 Model Description For a Single Cantilever

The design of a single cantilever sensor is shown schematically in Fig.1. It consists of two adjacent highly doped Si beams forming the two plates of a capacitor. One of the beams is rigid, while the other (hereafter referred to as the cantilever) is fairly soft and represents the movable part of the structure.

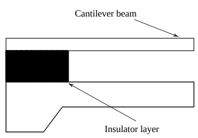

Figure 1: A schematic of an electrostatically driven cantilever.

If the length of the cantilever is much bigger than its distance from the bottom plate, the capacitance is given by

$$C(x) = \frac{\epsilon_o A}{d - z},$$

where $\epsilon_o = 8.85\ 10^{-12} As/Vm$ is the permittivity in vacuum, A is the area of the plates, d is the gap between them and z is the vertical displacement of the cantilever from its rest position.

If we apply a voltage $V(t)$, the attractive force, F_a, between the capacitor plates can be easily found to be

$$F_a = \frac{1}{2}\frac{\epsilon_o A}{d^2}\frac{V^2(t)}{(1-\frac{z}{d})^2} \approx \frac{1}{2}\frac{\epsilon_o A}{d^2}(1+2\frac{z}{d})V^2(t),$$

where the approximation holds when $\frac{z}{d} << 1$.

Hence, the equation of motion of the cantilever is described by

$$m\ddot{z} + kz = \frac{1}{2}\frac{\epsilon_o A}{d^2}(1+2\frac{z}{d})V^2(t), \qquad (1)$$

where $k = \frac{Ewt^3}{4L^3}$ is the spring constant of the cantilever, $E = 1.7\ 10^{11} N/m^2$ is the Young's modulus of silicon, and L, w, t are respectively length, width and thickness of the cantilever. In the simulations presented in this paper we have chosen the following values of the parameters: $L =$, $w =$, $t =$ and $d =$.

If we apply a sinusoidal voltage $V(t) = V_o \cos \omega_o t$, equation (1) can be rewritten, after few algebraic steps, as

$$z'' + (a - 2q\cos 2t)z = u_f(t), \qquad (2)$$

where the prime denotes the derivative with respect to the scaled time $\tau = \omega_o t$; $a = \frac{k}{m\omega_o^2} - \frac{1}{2}\frac{\epsilon_o A V_o^2}{md^3\omega_o^2}$, $q = \frac{\epsilon_o A V_o^2}{md^3\omega_o^2}$, and $u_f(t) = q\, d\cos^2(t)$.

Equation (2) is an instance of the Mathieu equation, a well-known and studied differential equation, that arises in boundary condition problems involving the wave equation. When $u_f(t) \equiv 0$, this equation has very peculiar stability properties, that have been extensively investigated. As a and q vary in $\mathbf{R}$, its stable solutions can be periodic, but they never decay to zero. In the case of our interest, where $u_f(t) \neq 0$ and periodic, we can prove that, for any pair of parameters a and q, the forced equation retains the same stability properties as the unforced one, with the only exception of the curves at the boundary between stable and unstable regions.

We consider the current generated as the output y of the system

$$y = i(t) = \frac{d}{dt}(CV) = \frac{\epsilon_o A}{d^2}\frac{V}{(1-\frac{x}{d})}\dot{x} + \frac{\epsilon_o A}{d}\frac{\dot{V}}{(1-\frac{x}{d})},$$

whose first order approximation is given by

$$y = c_1(t)z + c_2(t)z' + v_f(t), \qquad (3)$$

where $c_1(t) = -\frac{\epsilon_o A V_o w_o}{d^2}\sin t$, $c_2(t) = \frac{\epsilon_o A V_o}{d^2}\cos t$, and $v_f(t) = \frac{\epsilon_o A V_o w_o}{d}\sin t$.

Introducing the vector $x = [z \quad \dot{z}]^T$, we can derive from (2) and (3) the state space representation of the cantilever model

$$\begin{array}{c} x' = A(t)x + B(t)u_f(t) \\ y = C(t)x + v_f(t), \end{array} \qquad (4)$$

where $A(t) = \begin{bmatrix} 0 & 1/w_o \\ -a + 2q\cos 2t & 0 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C(t) = [c_1(t) \quad c_2(t)]$.

Note that (4) is a linear, time-varying and T-periodic model, with $T = 2\pi$. In the next section we will see how, using lifting and sampling, we can reduce the problem of designing an observer for (4) to an almost equivalent problem of observer design for a standard LTI model.

3 The Optimal Observer Problem

In this section we address the problem of designing a dynamical system capable of providing an estimate $\hat{x}$ for the cantilever displacement, based on the measurement of the current generated. This approach to sensing is particularly advantageous from the point of view of implementation, as it requires a simpler circuitry. As a matter of fact, the extraction of the desired information is left to a software elaboration of the measurements.

In the LFT framework, the observer problem can be formulated as an $\mathcal{H}_\infty$ filtering problem, by defining the variable $z = x - \hat{x}$ (estimation error), and considering the generalized plant (see Fig.2)

$$G_{gen} := \left[\begin{array}{c|cc} \text{A(t)} & [\text{M} \quad 0] & 0 \\ \hline \text{I} & 0 & \text{-I} \\ \text{C(t)} & [0 \quad \text{N}] & 0 \end{array}\right] = \left[\begin{array}{c|cc} \text{A(t)} & B_1 & 0 \\ \hline C_1 & 0 & D_{12} \\ C_2(t) & D_{21} & 0 \end{array}\right], \qquad (5)$$

where the exogenous input $w = [d \quad n]^T$ represents process and measurement noise, the matrices $A(t)$, $C(t)$ are as in (4) and the input $u = x$ is the output of the observer system. Notice that we don't need to account for the signals u_f and v_f in (4): since they are known, their presence does not affect the observer design.

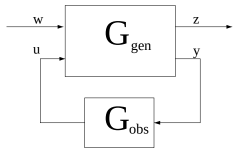

Figure 2: A schematic of the observer problem.

In this framework the optimal observer problem amounts at finding a dynamical system G_{obs} such that the $\mathcal{H}_\infty$ norm of the transfer function T_{zw} from w to z is minimized. If the system is time-invariant, and has the structure of (5), it turns out that G_{obs} is a Luenberger observer, whose gain L comes from the solution of an appropriate Riccati equation.

3.1 The Lifted System

The lifting technique is a very useful theoretical tool for dealing with periodic systems. The idea of the lifting is to associate with a T-periodic system G, an equivalent discrete time-invariant system $\hat{G}$. Intuitively speaking, this is done by decomposing the input and output signals of G into a sequence of segments, corresponding to the signals over successive intervals of length T. It can be proved (see [3] for instance) that this induces a rearrangement of the original T-periodic system G, such that its lifted equivalent $\hat{G}$ is shift invariant. In fact, there is a strong correspondence between a system and its lifting, that preserves not only algebraic system properties, such as cascade decomposition and feedback, but also internal stability and induced system norms. This latter property is of particular interest to us, in view of the observer problem we aim at solving.

Even though the equivalent lifted system has the much desirable properties of being linear, shift-invariant and norm preserving, it is infinite dimensional (since by construction its input/output spaces are infinite dimensional). Therefore, the observer design problem, as we have formulated it above, is not easily solvable, as it would involve an infinite dimensional minimization.

The approach we follow, along the lines of [1], is to convert this infinite dimensional minimization to an almost equivalent finite dimensional problem. This problem corresponds to the optimal observer design for the finite dimensional system obtained by fast sampling (5). Here, by almost equivalent we mean that the problem we finally solve is an approximation of the original one. It has been proved in [1] that this approximation converges at the rate of $1/N$, if T/N is the sampling period. Hence, by increasing the number of samples per period we can approximate the optimal solutions of the original system to any prescribed degree of accuracy (see [1]).

In [1], analytical expressions to compute the system matrices corresponding to the approximate problem are provided. Unfortunately, these formulas require the computation of the state transition matrix of the original system. This problem, in general and with very few exceptions, is not solvable if the system is time-varying. The Mathieu equation is not such an exception. However, rearranging the terms in equations (5), and introducing the fictitious output

$$y_o = [1 \quad 0]x = C_o x,$$

we can isolate the time-invariant part of the state equation in (5) and view its time-varying part as a feedback from the output y_o

$$\begin{array}{rcllll} x' & = & A_o x & +B_1 w & & +B_3 v \\ z & = & C_1 x & & +D_{12} u & \\ y & = & C_2(t) x & +D_{21} w & & \\ y_o & = & C_o x & & & \\ v & = & -K(t) y_o, & & & \end{array} \qquad (6)$$

where $A_o = \begin{bmatrix} 0 & 1/w_o \\ -a & 0 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $K(t) = 2q\cos 2t$. The advantage is that rewriting the system in this form we are able to compute the state space representation.

If we denote with $\underline{s}_k$ the N dimensional vector containing the N samples of $s(t)$ corresponding to the k-th time period, $[kT, \quad (k+1)T)$

$$\underline{s}_k := \begin{bmatrix} s(kT) \\ s(kT + \frac{T}{N}) \\ s(kT + \frac{2T}{N}) \\ \vdots \\ s((k+1)T - \frac{T}{N}) \end{bmatrix},$$

the state equations of the approximate problem (lifted and sampled) corresponding to (6) turn out to be

$$\begin{array}{rclllll} x_{k+1} & = & \underline{A} x_k & +\underline{B}_1 \underline{w}_k & & +\underline{B}_3 \underline{v}_k \\ \underline{z}_k & = & \underline{C}_1 x_k & +\underline{D}_{11} \underline{w}_k + & \underline{D}_{12} \underline{u}_k & +\underline{D}_{13} \underline{v}_k \\ \underline{y}_k & = & \underline{C}_2 x_k & +\underline{D}_{21} \underline{w}_k & & +\underline{D}_{23} \underline{v}_k \\ \underline{y}_{ok} & = & \underline{C}_o x_k & +\underline{D}_{o1} \underline{w}_k & & +\underline{D}_{02} \underline{v}_k \\ \underline{v}_k & = & -\underline{K}\, \underline{y}_{ok}, & & & \end{array} \qquad (7)$$

where the presence of the new D_{ij} matrices is a result of the lifting. The analytical expression of all the matrices in (7) can be computed following the indications given in [1], with the exception of those coming from the time-varying part of the system (equations 3-5 in 7), which are

$$\underline{C}_2 = \begin{bmatrix} C_0 & e^{A_o^T T_s} C_{\frac{1}{N}}^T & e^{2A_o^T T_s} C_{\frac{2}{N}}^T & \cdots e^{A_o^T T_s (N-1)} C_{1-\frac{1}{N}}^T \end{bmatrix}^T$$

$$\underline{D}_{21} = \begin{bmatrix} D_{21} & 0 & 0 & \cdots & 0 \\ C_{\frac{1}{N}} \hat{B}_1 & D_{21} & 0 & \cdots & 0 \\ C_{\frac{2}{N}} e^{A_o T_s} \hat{B}_1 & C_{\frac{2}{N}} \hat{B}_1 & D_{21} & \ddots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ C_{1-\frac{1}{N}} e^{A_o T_s (N-2)} \hat{B}_1 & \cdots & & C_{1-\frac{1}{N}} \hat{B}_1 & D_{21} \end{bmatrix}$$

$$\underline{D}_{23} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ C_{\frac{1}{N}} \hat{B}_3 & 0 & 0 & \cdots & 0 \\ C_{\frac{2}{N}} e^{A_o T_s} \hat{B}_3 & C_{\frac{2}{N}} \hat{B}_3 & 0 & \ddots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ C_{1-\frac{1}{N}} e^{A_o T_s (N-2)} \hat{B}_3 & \cdots & & C_{1-\frac{1}{N}} \hat{B}_3 & 0 \end{bmatrix}$$

and

$$\underline{K} = \begin{bmatrix} 2q & 0 & \dots & & 0 \\ 0 & 2q\cos(\frac{T}{N}) & 0 & \dots & 0 \\ 0 & 0 & 2q\cos(\frac{2T}{N}) & & 0 \\ \vdots & & \dots & \ddots & \\ 0 & & \dots & & 2q\cos(\frac{N-1}{N}T) \end{bmatrix},$$

with

$$e^{A_o T_s} = \begin{bmatrix} \cos\sqrt{a}T_s & \frac{1}{\sqrt{a}}\sin\sqrt{a}T_s \\ -\sqrt{a}\sin\sqrt{a}T_s & \cos\sqrt{a}T_s \end{bmatrix},$$

$C_k = C_2(KT)$ and $\hat{B}_j$ is the sampled matrix corresponding to B_j in (6). Finally, by defining the following matrices

$$\begin{aligned} F &= \underline{A} - \underline{B}_3\ \underline{K}(I + \underline{D}_{o2}\underline{K})^{-1}\underline{C}_o, \\ G_1 &= \underline{B}_1 - \underline{B}_3\ \underline{K}(I + \underline{D}_{o2}\underline{K})^{-1}\underline{D}_{o1}, \\ H_1 &= \underline{C}_1 - \underline{D}_{13}\ \underline{K}(I + \underline{D}_{o2}\underline{K})^{-1}\underline{C}_o, \\ H_2 &= \underline{C}_2 - \underline{D}_{23}\ \underline{K}(I + \underline{D}_{o2}\underline{K})^{-1}\underline{C}_o, \\ J_{11} &= \underline{D}_{11} - \underline{D}_{13}\underline{K}(I + \underline{D}_{o2}\underline{K})^{-1}\underline{D}_{o1}, \\ J_{21} &= \underline{D}_{21} - \underline{D}_{23}\underline{K}(I + \underline{D}_{o2}\underline{K})^{-1}\underline{D}_{o1}, \end{aligned}$$

and $J_{12} = \underline{D}_{12}$, the generalized plant associated to the approximate problem is given by

$$G_{apprx} := \left[\begin{array}{c|cc} \text{F} & G_1 & 0 \\ \hline H_1 & J_{11} & J_{12} \\ H_2 & J_{21} & 0 \end{array}\right], \qquad (8)$$

and describes a finite dimensional discrete time-invariant system. For this system we can apply known results from classical system theory.

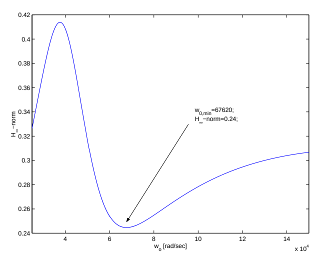

Figure 3: $\mathcal{H}_\infty$−norm vs. frequency of excitation.

Figure (3) describes the dependence of the closed loop $\mathcal{H}_\infty$ norm from the frequency of excitation, ω_o. The range of frequencies considered is chosen to be around the resonant frequency of the cantilever, $\omega_r = 51kHz$. Based on the analysis of this plot, we have chosen $\omega_o = 67620$, which corresponds to a minimum value in the optimal performance index ($\mathcal{H}_\infty - norm = 0.24$). In the next section we will see how to use this information to design a suboptimal reduced order observer.

4 The reduced order observer

The idea behind a reduced order observer is to use the information about the state of the system that is provided by the output signal and leave to the observer the task of estimating a smaller portion of the state vector. We refer the interested reader to any book on linear systems theory for the details of this standard technique. Here we just want to provide the expression of the matrix T^{-1} that defines the required change of coordinates,

$$T^{-1} = \begin{bmatrix} H(t) \\ C(t) \end{bmatrix} = \begin{bmatrix} c_1^{-1}cost & -c_2^{-1}sint \\ c_2 sint & c_1 cost \end{bmatrix},$$

where c_i, $i = 1, 2$ are the constant coefficients of $c_i(t)$ in (4), $T^{-1} \in \mathbf{C}^1$ and $det(T^{-1}) = 1$ at each t. Notice that to be well-defined, this change of coordinates requires to consider a 'noiseless' output, i.e. $\tilde{y} = C(t)x$.

The equations of the observer turn out to be

$$\begin{aligned} \dot{\hat{v}} &= (A_{11}(t) + L(t)A_{21}(t))\hat{v} + M(t)y \\ \hat{x} &= T(t)\begin{bmatrix} \hat{v} - L(t)y \\ y \end{bmatrix}, \end{aligned}$$

where A_{11}, A_{21}, M are π-periodic matrices that can be computed from the system matrices in (4). $L(t)$ is the design parameter, through which we can influence the behavior of the observer.

The state estimation error, $e_x = x - \hat{x}$ is described by

$$e_x = T\begin{bmatrix} e_v \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & D_x(t) \\ 0 & 0 & \end{bmatrix} w, \qquad (9)$$

where e_v is governed by the equation $\dot{e}_v = (A_{11} + LA_{21})e_v + B_v(t)w$ and the matrices D_x,B_v are known functions of the system matrices. L needs to be chosen so that (9) is asymptotically stable. For a T-periodic system this is equivalent to say that its characteristic multipliers, which are the eigenvalues of the state transition matrix computed at T, are in norm less than 1, $|\lambda(\Phi(T))| < 1$. Since (9) is scalar, $\Phi(T)$ can be easily computed

$$\Phi(T) = e^{\int_0^T (A_{11} + LA_{21})(\sigma)d\sigma},$$

and the condition on the characteristic multipliers is equivalent to the condition $\int_0^T (A_{11} + LA_{21})(\sigma)d\sigma < 0$. In particular, we have

$$A_{11} + LA_{21} = d_1\sin(2t) + d_2\sin(4t) + L(t)[d_3 + d_4\cos(2t) + d_5\cos(4t)],$$

where d_i are known functions of the system parameters. By taking $L(t) = k\cos(2t + \phi)$ the stability condition becomes $kd_3\pi\cos\phi > 0$, which poses a constraint on the choice of k and ϕ. For instance, for the value of the parameters we have considered so far in our simulations, it turns out that we must have

$$k\cos(\phi) < 0.$$

Figure (4) shows simulation results for two values of k and $\phi = 0$, in the absence of noise: as expected the error dynamics are asymptotically stable.

However, we want to select the parameters of this observer not only to ensure stable error dynamics, but also to optimize its performance, with the $\mathcal{H}_\infty$-norm as its measure. Figure (5) depicts the value of this norm as k and ϕ vary in $\mathbf{R}^+$ and $[0\ \ 2\pi)$ respectively. Based on this plot, a better informed choice of k and ϕ turns out to be $k = 0.001$ and $\phi = 3.63$, which give $\mathcal{H}_\infty$-norm=45.

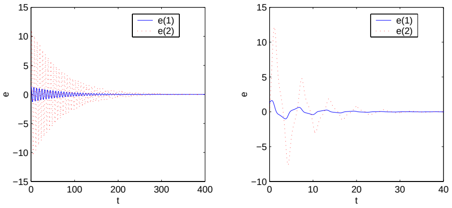

Figure 4: Estimation error for different values of the observer gain k ($\phi = 0$).

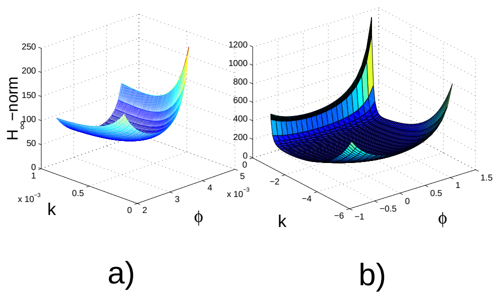

Figure 5: Estimation error for different values of the observer gain k ($\phi = 0$): a) $k > 0$ $\cos(\phi) < 0$, b) $k < 0$ $\cos(\phi) > 0$.

5 Array of Microcantilevers

In this section we investigate the properties and derive a mathematical model for the parallel connection of electrostatically actuated microcantilevers.

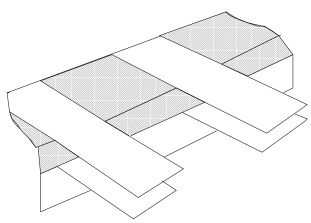

Figure 6: A schematic of the multicantilever array.

In this configuration, the bottom rigid plates of each cantilever-capacitor are connected to a common base, as shown schematically in Fig.(6), and all cantilevers are driven at the same voltage. Though each cantilever is independently actuated, its dynamics are influenced by the electric fringing fields generated by the capacitors nearby. As a consequence, the model we introduced for the single cantilever has to be modified to take into account this interaction.

In the model we propose, we consider that the cantilevers exert a repulsive force on each other, since they are all positively charged. In this work we describe this interaction via a point charge model.

The idea is shown schematically in Fig.(7), that depicts only the case of two capacitors. Each cantilever is represented as a charged particle, $q_i = C_i V_i$, and the mutual interaction is

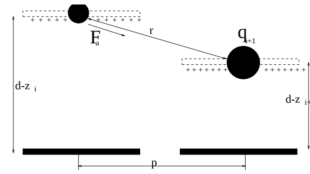

Figure 7: A schematic of the coupling capacitance model.

described by Coulomb's law

$$\begin{aligned} F_{ij} &= \frac{1}{4\pi\epsilon_o}\frac{q_i q_j}{r^2} \\ &= \frac{\epsilon_o A^2}{4\pi}\frac{V^2}{(d-z_i)(d-z_j)[p_{ij}^2+(z_i-z_j)^2]}, \end{aligned}$$

where $p_{ij} = |i-j|p$ is the pitch distance between the i-th and j-th capacitors, and z_i is the vertical displacement of the i-th cantilever. We assume that the lateral stiffness of the cantilevers is high enough to prevent any lateral motion, so that the only component of the force that really affects their behavior is the vertical, whose first order approximation is

$$F_{ij}^{\perp} = c_{ij}(z_i - z_j)V^2, \tag{10}$$

with $c_{ij} = \frac{\epsilon_o A^2}{4\pi d^2 p_{ij}^3}$.

Note that, due to the symmetry of the array, the coefficients c_{ij} are even functions of j, i.e. $c_{i,j} = c_{i,-j}$. Moreover, as it is reasonable to expect from a physical argument, their value decays to zero as j tends to infinity.

Taking into account this coupling force (10), the state equations for the i-th cantilever become

$$\begin{aligned} x_1(t,i)' &= \tfrac{1}{w_o}x_2(t,i) \\ x_2(t,i)' &= (-\hat{a} + 2\hat{q}\cos 2t)x_1(t,i) + u(t,i) + \\ &\quad + \textstyle\sum_{j\neq i}\left[a_{ij} - a_{ij}\cos 2t\right]x_1(t,j), \end{aligned} \tag{11}$$

where i is the spatial variable introduced to denote the cantilevers in the array, $a_{ij} = \frac{V_o^2}{2w_o}c_{ij}$, $\hat{a} = a + \bar{a}$, $\hat{q} = q - \frac{\bar{a}}{2}$, with $\bar{a}$ sum of the convergent series $\bar{a} = \sum_{j\neq i} a_{ij}$.

Equations (11) provide a local description of the system, where by local we mean limited to the i-th cantilever. However, no term in equations (11) is specific to the i-th cantilever, that is, modulo a shift in the spacial index, these equations describe the dynamical behavior of any cantilever in the structure. Systems that satisfy this property are called *spatially-invariant* [5] and in our case, more precisely, *distributed spatially-invariant*, since all the elements of the array are actuated and sensed independently.

It has been shown that, by applying the Fourier transform in the spatial domain, which for a two-dimensional signal is defined as

$$V(\lambda, t) = \sum_{k=-\infty}^{\infty} v(k,t)e^{-ik\lambda} \quad ,$$

it is possible to associate a two dimensional distributed system with a one dimensional parametric system, which is equivalent to the former, but that can be analyzed using well known results from classical systems theory. We refer the interested reader to [5] for the main results concerning this approach in the study of spatially invariant distributed systems.

As we pointed out before, the physical and, consequently, the mathematical structure of the multicantilever model allows us to embed it in the class of spatially invariant systems. This means that in order to study the multicantilever, we do not need to deal with the infinite dimensional model corresponding to the complete stucture, but that we can use instead the parametrized local model

$$x(t,\lambda)' = \begin{bmatrix} 0 & \frac{1}{w_o} \\ -a(\lambda) + 2q(\lambda)\cos 2t & 0 \end{bmatrix},$$

where $a(\lambda) = \hat{a} - \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} a_k e^{-ik\lambda}$ and $q(\lambda) = \hat{q} - \frac{1}{2}\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} a_k e^{-ik\lambda}$.

At this point, we can proceed in a way which is formally analogous to the single cantilever case. A detailed analysis and results, both theoretical and simulation, are the subject of our present research.

6 Conclusions

In this paper we have derived a mathematical model for an electrostatically actuated microcantilever, considered both as a single unit sensor/actuator and as an element of an array architecture. In both cases, the microcantilever constitutes the movable plate of a capacitor and its displacement is controlled by the voltage applied across the plates. The current generated is used as the sensing signal. In the case of a single cantilever, its dynamics are regulated by a special second order differential equation with periodic coefficients, the Mathieu equation. As for the array configuration, we have shown that the system is an example of a spatially invariant system with distributed control and sensing, property which will be used in the design of a distributed observer for the cantilevers displacements. We have formulated the optimal observer problem for the single cantilever and shown that its solution can be approximated to any prescribed degree of accuracy by solving an almost equivalent problem for a standard LTI discrete system, obtained by lifting and fast-sampling the original periodic system. This design has been used to select the frequency of excitation that makes our model more easily observable. Moreover, it has been used as a benchmark to compare the performance of a reduced order observer and tune its parameters. The extension of these results to the array configuration is the subject of our current research.

Acknowledgements

This work partially supported by a DOD MURI grant for "uncertainty managment in complex systems".

References

[1] B. Bamieh, M.A. Dahleh and J.B. Pearson,"Minimization of the L^∞-Induced Norm for Sampled-Data Systems," *IEEE Trans. on AC*, Vol. 38, No. 5, pp. 717-732, May 1993.

[2] N. Blanc, J. Brugger, N.F. de Rooji and U. Durig, "Scanning Force Microscopy in the Dynamic Mode Using Microfabricated Capacitive Sensors", *J. Vac. Sci. Technol. B*, 14(2), pp.901-905, Mar/Apr 1996.

[3] T. Chen and B. Francis, "Optimal Sampled-Data Control Systems", Springer, 1995.

[4] B.W. Chui, T.W. Kenny, H.J. Mamin, B.D. Terris and D. Rugar, "Independent Detection of Vertical and Lateral Forces with a Sidewall-Implanted Dual-axis Piezoresistive Cantilever", *Appl. Phys. Lett.*, Vol. 72, No.11, pp.1388-1390, March 1998.

[5] E.W. Kamen, "Stabilization of Linear Spatially-Distributed Continuous-Time and Discrete-Time Systems", in *Multidimensional Systems Theory*, Chap.4, N.K. Bose (ed.), D. Reidel Publishing Company, 1985.

[6] M. Tortonese, R.C. Barrett and C.F. Quate, "Atomic Resolution with an Atomic Force Microscope Using Piezoresistive Detection", *Appl. Phys. Lett*, Vol. 62, pp. 834-836, 1993.

[7] P. Gaucher, D. Eichner, J. Hector and W. von Munch, "Piezoelectric Bimorph Cantilever for Actuationf and Sensing Applications", *J. Phys. IV France*, Vol. 8, pp. 235-238, 1998.

[8] T. Itoh, T. Ohashi and T. Suga, "Piezoelectric Cantilever Array for Multiprobe Scanning Force Microscopy", *Proc. of the IX Int. Workshop on MEMS*, San Diego, CA, USA, 11-15 Feb. 1996. Mew York, MY, USA: IEEE, 1996. pp. 451-455.

[9] S.C. Minne, S.R. Manalis and C.F. Quate, "Parallel Atomic Force Microscopy Using Cantilevers with Integrated Piezoresistive Sensors and Integrated Piezoelectric Actuators", *Appl. Phys. Lett.*, Vol. 67, No. 26, pp. 3918-3920, 1995.

[10] Q.A. Huang and N.K.S. Lee, "A Simple Approach to Characterizing the Driving Force of Polysilicon Laterally Driven Thermal microactuators", *Sensors and Actuators*, 80, pp. 267-272, 2000.

[11] P. Attia, M. Boutry, A. Bosseboeuf and P. Hesto, "Fabrication and Characterization of Electrostatically Driven Silicon Microbeams", *Microelectronics J.*, 29, pp. 641-644, 1998.

[12] B. Bamieh, F. Paganini and M.A. Dahleh, "Optimal Control of Distributed Actuator and Sensor Arrays", to appear in *Proc. SPIE 5'th Ann. Intl. Symp. on Smart Structures and Materials*, San Diego, March 1998.

[13] B. Bamieh, "The Structure of Optimal Controllers of Spatially-Invariant Distributed Parameter Systems", in *Proc. 36'th IEEE Conference on Decision and Control*, San Diego, CA, December, 1997.

[14] B. Bamieh, F. Paganini and M.A. Dahleh, "Distributed Control of Spatially Invariant Systems", submitted to the *IEEE Trans. Automatic Control*, May 1998.