MAT 202:

Introduction To Mathematics For DSP.

Set 6: Frequency Selective Filters.

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A filter is a device that discriminates according to some attribute of the input. In case of signals, frequency selective filters pass signals with frequency components in some bands and attenuate signals with frequency components in other bands. In signal processing, filtering is used in a variety of ways, like removing undesired noise from desired signals, spectral shaping, spectral analysis of signals and so on.

I] DEFINITION:

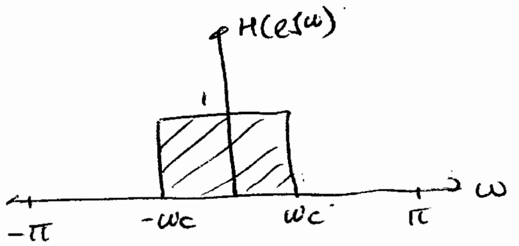
If $X(e^{j\omega})$ is the input to a filter with frequency response $H(e^{j\omega})$, then the output of the filter is given by

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}).$$

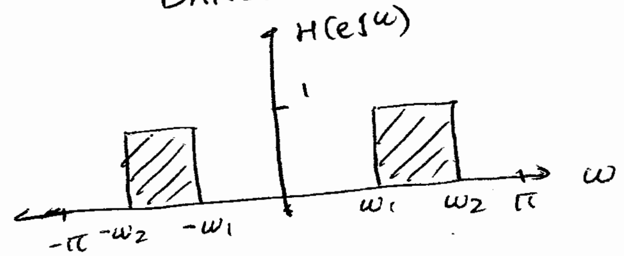
Any linear time-invariant (LTI) system can be treated as a frequency shaping filter.

II] IDEAL FILTER CHARACTERISTICS:

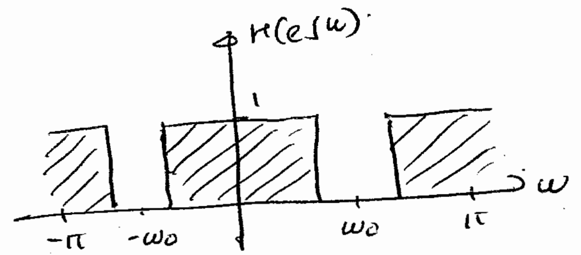
LOWPASS



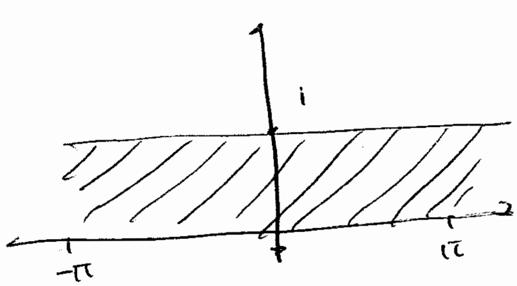
BANDPASS



HIGHPASS



ALL PASS



An ideal filter should have
1) Constant magnitude characteristic
2) Linear phase characteristic

$\therefore H(\omega) = |H(\omega)| e^{\measuredangle H(e^{j\omega})}$

$\therefore |H(e^{j\omega})| = 1 \quad \omega_1 < \omega < \omega_2$

$\arg(H(e^{j\omega}))$ = linear function of ω.

In practice, an ideal filter cannot be realized. The ideal filter response is finite in the frequency domain. Hence it requires an infinite impulse response in the ~~frequency~~ time domain, which cannot be obtained in practice. However, the ideal filter characteristics can be approximated very closely by practical, physically realizable filters.

In the following section, we design a few filters based on pole-zero placement. The basic principle of pole-zero placement is to place the poles near points on the unit circle corresponding to frequencies to be emphasized, and to place zeros near frequencies to be de-emphasized. Further, the following constraints must be imposed

1) All poles must be inside the unit circle for the filter to be stable
2) All complex poles and zeros must occur in complex-conjugate pairs, for the filter co-efficients to be real.

III] TYPES OF FILTERS :

A] LOW PASS FILTER.

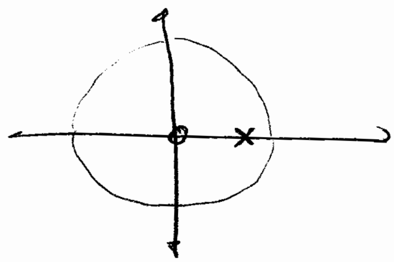
For designing a LPF, the poles should be placed near the unit circle at points corresponding to low frequencies (near $\omega = 0$), and zeros should be placed near or on the unit circle at points corresponding to high frequencies (near $\omega = \pi$).

e.g: $H_1(z) = \dfrac{1-a}{1-az^{-1}}$ (single pole)

$H_2(z) = \dfrac{1-a}{2} \dfrac{1+z^{-1}}{1-az^{-1}}$ (one pole, one zero).

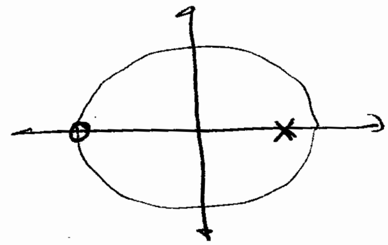
$|H_{LP}(e^{j0})| = 1 \; ; \quad |H_{LP}(e^{j\pi})| = 0$

$H_1(z)$



z-plane

$H_2(z)$



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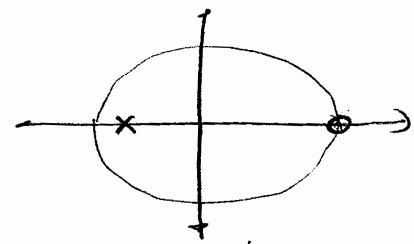
B] HIGHPASS FILTER.

The poles should be placed near the unit circle at points corresponding to $\omega = \pi$ (high frequencies), and zeros should be placed near or on the unit circle at points corresponding to $\omega = 0$ (low frequencies).

e.g: $H_{HP}(z) = \dfrac{1-a}{2} \cdot \dfrac{1-z^{-1}}{1+az^{-1}}$



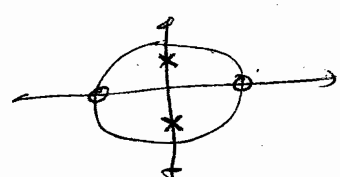
z-plane

$|H_{HP}(e^{j0})| = 0$; $|H_{HP}(e^{j\pi})| = 1$

C] BANDPASS FILTER:

A bandpass filter should have one or more pairs of complex conjugate poles near the unit circle, in the vicinity of the frequency band that constitutes the passband of the filter and zeros at $\omega = 0$ & $\omega = \pi$.

e.g: $H_{BP}(z) = \dfrac{G(z-1)(z+1)}{(z-jr)(z+jr)}$

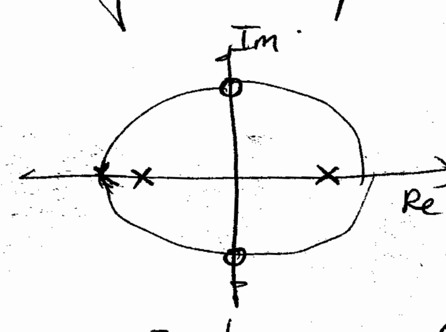


z-plane

D] BANDSTOP FILTER.

A bandstop filter should have one or more pairs of complex conjugate pairs of zeros in the vicinity of the frequency band to be ignored and poles near the unit circle at $\omega = 0$ & $\omega = \pi$.

e.g: $H_{BS}(z) = G\,\dfrac{(z-j)(z+j)}{(z-a)(z+a)}$



z-plane