Statistical mechanical ensembles

property	microcanonical	canonical	grand canonical	isothermal-isobaric
constant conditions	E, V, N	T, V, N	T, V, μ	T, P, N
fluctuations	none	E	E, N	E, V
microstate probabilities	$\wp_m = \frac{\delta_{E_m=E}}{\Omega(E,V,N)}$	$\wp_m = \frac{e^{-\beta E_m}}{Q(T,V,N)}$	$\wp_m = \frac{e^{-\beta E_m + \beta\mu N_m}}{\Xi(T,V,\mu)}$	$\wp_m = \frac{e^{-\beta E_m - \beta P V_m}}{\Delta(T,P,N)}$
partition function	$\Omega(E,V,N) = \sum_n \delta_{E_n=E}$	$Q(T,V,N) = \sum_n e^{-\beta E_n}$	$\Xi(T,V,\mu) = \sum_N \sum_n e^{-\beta E_n + \beta\mu N}$	$\Delta(T,P,N) = \sum_V \sum_n e^{-\beta E_n - \beta P V}$
relations to other partition functions	---	$Q = \sum_E e^{-\beta E}\Omega$	$\Xi = \sum_N \lambda^N Q$ $= \sum_N \sum_E \lambda^N e^{-\beta E}\Omega$ $\lambda \equiv \exp[\beta\mu]$	$\Delta = \sum_V e^{-\beta P V} Q$ $= \sum_V \sum_E e^{-\beta E - \beta P V}\Omega$
thermodynamic potential	$S = k_B \ln \Omega(E,V,N)$	$A = -k_B T \ln Q(T,V,N)$	$PV = k_B T \ln \Xi(T,V,\mu)$	$G = -k_B T \ln \Delta(T,P,N)$
classical partition function	$\Omega = \frac{1}{h^{3N} N!}\int \delta[H(\mathbf{p}^N, \mathbf{r}^N) - E]\, d\mathbf{p}^N d\mathbf{r}^N$	$Q = \frac{Z(T,V,N)}{\Lambda(T)^{3N} N!}$ $Z \equiv \int e^{-\beta U(\mathbf{r}^N)} d\mathbf{r}^N$ $\Lambda \equiv (h^2/2\pi m k_B T)^{\frac{1}{2}}$	$\Xi = \sum_{N=0}^{\infty} \frac{\lambda^N Z(T,V,N)}{\Lambda(T)^{3N} N!}$ $\lambda \equiv \exp[\beta\mu]$	$\Delta = \frac{1}{\Lambda(T)^{3N} N!}\int_0^\infty e^{-\beta P V} Z(T,V,N) dV$

**Sums over n correspond to sums over all microstates at a given V and N.
**Sums over N are from 0 to ∞, for V from 0 to ∞, and for E from $-\infty$ to ∞.