Pressure-Dependent Damping Characteristics of Micro Silicon Beam Resonators for Different Resonant Modes

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Abstract: This paper is to investigate (experimentally and model-based) the resonant-mode dependence of micro cantilevers' damping characteristics and how this dependence changes for different pressure ranges: air pressure (100Torr and higher), high vacuum (~1mTorr) and pressures in between. In air pressure, we focus on the damping caused by the fluid drag force. A new frequency dependent fluid damping model is proposed and experimentally validated. At high vacuum, thermo-elastic damping (TED) is demonstrated to be the main damping source experimentally, for frequency range from 10Khz to 2MHz and up to 6 resonant modes. By proper design we are able to tell that the "effective" viscous damping is dominant in the transition pressure regime.

Keywords: fluid damping, drag force, Q factor, TED

INTRODUCTION

There are two general approaches to escalate device resonant frequencies: Decreasing the device dimensions or operating at higher resonant modes to keep the device "bulky". One of the disadvantages of the first option is the relatively small Q factor one can get due to high surface effects to oscillator volume [1]. Higher-mode oscillation can partially overcome this because of the relatively lower surface to bulk ratio. Applications of using higher modes include high mode cantilever based mass sensors, AFM and FBAR [2]. For both approaches, understanding how the damping varies with the resonant frequencies and the resonant modes is very important to these applications. This dependence will be different for different damping mediums. In this research, we focus on a wide pressure spectrum, from high vacuum to air pressure.

Generally the whole pressure range can be divided into the following regimes correlated to the dominant damping mechanism.

- High vacuum regime: Intrinsic damping
- Molecular regime: Molecular damping
- Transition regime: Rarefied gas damping
- Viscous regime: Fluid damping

The boundaries between each regime are transitional, continuous and not easily defined, especially between the molecular regime and transition regime. One of the most common approaches to separate these regimes is to use Knudsen number K_n, which is defined as the ratio of the molecular mean free path to the characteristic dimension of the device [3].

Much research has investigated the pressure dependence of the MEMS devices for certain geometries [3-4], but little has been explored regarding the resonant frequency dependence quantitatively. This incomplete knowledge of the frequency dependence of the damping characteristics not only limits the design of the resonators with specific damping requirements [5], but also obstructs the further improvement of devices' functionalities by proper resonant mode choices.

This paper is organized in the following way: First we present the devices tested for this research and the way we define the damping. Second, we discuss the damping in air pressure regime with the proposal and the validation of a new frequency dependent fluid damping model, followed by the discussion of damping in high vacuum. Last part is about the damping in the transition regime, where we experimentally support the "effective" viscous damping mechanism.

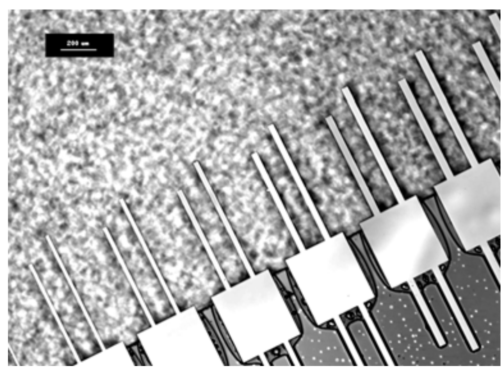

Figure 1 Cantilevers without substrate

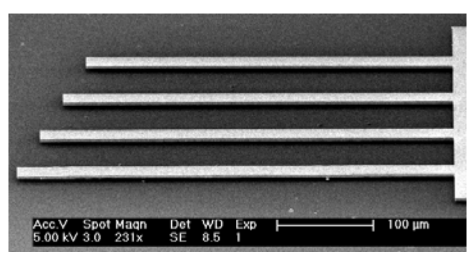

Figure 2 Cantilevers with substrate

DEVICES AND DAMPING

The fabrication methods of devices, either surface or bulk machining, will impact geometric neighborhood of the resonator space. This can have an impact on the fluid motion displaced by the device oscillation. To explore these variations, two kinds of cantilevers (see Figures 1-2) are tested in this research, cantilevers with and without adjacent substrates. The devices are made from the SOI wafers and actuated either electro-statically (with substrate) or base-excited piezo-electrically (without substrate). A laser-vibrometer setup [6] is used for the characterization.

The general equation of motion for a cantilever is:

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} + C\frac{\partial y}{\partial t} = p(x,t) \quad (1)$$

where $y = y(x,t)$ is the displacement, C is the damping coefficient, m is the mass per unit length, p is the external force. For fluid damping, term $C\frac{\partial y}{\partial t}$ denotes the drag force per unit length. To separate the spatial and temporal variables, we rewrite $y(x,t)$ as:

$$y(x,t) = \sum_{j=1}^{\infty}\phi_j(x)q_j(t) \quad (2)$$

where $\phi_j(x)$ are the normalized orthogonal modes of the cantilever. We have:

$$\ddot{q}_j + \frac{C}{m}\dot{q}_j + \omega_j^2 q_j = \frac{1}{m}\int\phi_j(x)p(x,t)dx = p_j^*(t) \quad (3)$$

Besides the damping coefficient C, other two damping parameters (β and Q factor) are also used frequently with the following relations:

$$2\beta_j = \omega_j / Q_j = C_j / m \quad (4)$$

FLUID DRAG FORCE AND AIR PRESSURE DAMPING AND

Air pressure is a very desirable operating regime because of the real world sensing applications and opportunities and is therefore important to develop an according damping design methodology. Here we focus on the damping of objects freely oscillating in fluid with infinite boundary. For cantilevers, it means the vibratory cantilevers without substrate. In this case, the drag force by the fluid is the dominant damping source for the objects.

Model: Our high quality factor resonators enable a wide range of frequency dependent data to be collected because even in high viscous damping regions, air pressure, device harmonics are measurable using our laser vibrometer measurement system. Permutations in device geometry can identify trends in damping relationships. Using past work in this area as a guide we will unify some previous physical descriptions by experimental methods. It has been shown conclusively that the damping factor of a 3-dimensional object in a viscous fluid depends not only on device geometries but also on oscillatory frequencies [7]. For objects with simple geometries, such as sphere and ellipsoid, the analytical results can be derived modeling with a Stokes flow. For objects with infinite length, the resulting 2-D problem seems to be more difficult due to the fact that the Stokes flow assumption has no solution which satisfies the boundary conditions at the surface of the objects and infinite boundaries at the same time [7]. For vibratory cantilevers, we offer a damping model similar in concept to the damping of a sphere. It is a combination of result of a steady state flow and result of an oscillatory model. Our new model assumes the damping drag force per unit length is linearly dependent on $\sqrt{\omega}$. For cantilevers with a rectangular cross-section, our model also assumes the linear dependence of the damping drag force on the widths of the cantilevers. So the damping coefficient is linearly dependent on $width * \sqrt{\omega}$. Using a dimensionless parameter λ to encapsulate the frequency dependence of operation,

$$\lambda = \frac{Width}{Penetration\ Depth} = \frac{Width}{\delta} = \frac{Width * \sqrt{\omega}}{\sqrt{2\mu/\rho}} \quad (5)$$

where μ is the viscosity and ρ is the density, the damping coefficient of a cantilever has the form:

$$C = \pi\mu(a + b\lambda) \quad (6)$$

a and b are dimensionless cross-section-shape dependent constants which can be determined analytically or by numerical simulation [8,9]. The first term in (6) is frequency independent, implying the steady flow result. Oseen's drag force result of an elliptic cylinder is used as an approximation [8]:

$$a = \frac{4}{R_1/(R_1+R_2) - \gamma - \ln\left(k(R_1+R_2)/4\right)} \quad (7)$$

Here $\gamma \cong 0.577$ is the Euler constant, $2R_2$ is the width of the cantilever, $2R_1$ is the thickness, $k = \rho u/(2\mu)$, ρ is the density and u is the maximum velocity.

Testing Results: In Figure 3 we show the testing results for a set of cantilevers with different widths (20-50um), 600um length and 5um thickness. The figure show a clear linear dependence of the damping on the widths.

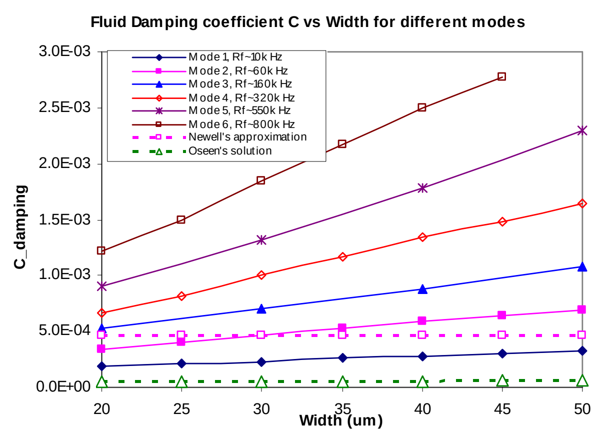

Figure 3 Damping coefficient for different width and resonant modes (600um long, 5um thick)

The results of equation (6) with a =0.5 b =2 are compared to the experimental results shown in Figure 3. And the comparison is shown in Figure 4 with respect to the frequency relative parameter λ for up to 6 resonant modes. It shows a good conformance between the experiments and our proposed model.

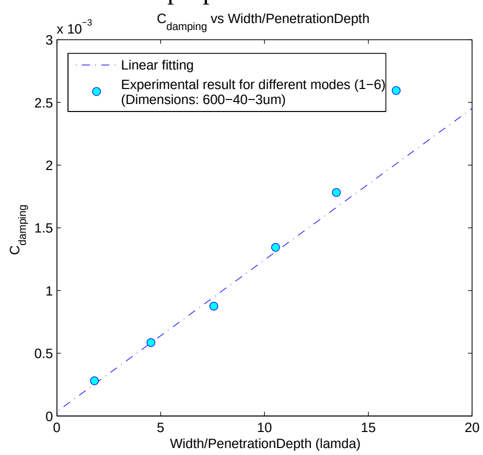

Figure 4 Comparison of the experimental results to the estimated results given by Equation (6) (@760Torr)

Above we discussed the damping caused by the fluid drag force for cantilevers without substrate. For cantilevers with substrate (see Figure 2), our testing results (not shown here) agree with the predicted damping caused by the squeezed film. According to W.S Griffin's squeeze film damping theory [10], damping coefficient C is independent of the resonant frequency and the resonant mode, with the assumption that the resonant frequency is much smaller than the "cut-off" frequency.

For pressure range from 100Torr to 760Torr, the damping given by model (6) all show a good conformance to the experimental damping result. Next, we will move to the discussion of the damping characteristics at the other end of the pressure range---high vacuum (~1mT).

INTRINSIC DAMPING AND TED

High vacuum testing often demonstrates the most interesting physical phenomena and enables the highest sensitivity measurements based on harmonic sensing. In high vacuum, the damping caused by the pressure contributes less to the overall damping comparing to other damping sources [1]. Other damping sources such as the intrinsic damping, anchor loss and surface damping become more important. At low pressure, the high Q factor we achieve (up to 2e5) enable us to explore the dominant intrinsic damping. The operational pressure minimum of the experimental environment is ~3mT. The devices we test here are the cantilevers with substrate (5um gap, see Figure 2). Other dimensions are: 5um thickness, 20um width and 100-600 length. Up to 6 resonant modes are tested and shown in Figure 5. By comparison, the analytical thermoelastic damping (TED) results [11] are also plotted in Figure 5. From the figure, we can see that the damping increases with the increasing of the resonant frequencies and resonant modes. The tested damping is close the TED, especially for the high resonant frequencies and higher modes. The error is less than 50% for frequency higher than 500 KHz. For low frequencies or low resonant modes, although the tested Q is in the same order with Q of TED, the difference is still considerable. The reason lies in the fact that for the low frequencies, the pressure damping still has an effect even at ~3mTorr. This will be demonstrated later.

According to Zener's approximation [11], the Q factor caused by TED is:

$$Q_{TED} = \left(\frac{C_{sp}\rho}{E\alpha^2 T_0} \right) \frac{\omega_D^2 + \omega^2}{\omega\omega_D}, \quad \omega_D = \left(\frac{\pi}{d} \right)^2 \frac{\kappa}{C_{sp}\rho} \quad (11)$$

Where ρ is the density, E is the Young's modulus, α is the thermal expansion coefficient and κ is the thermal conductivity, C_{sp} is the specific heat. ω_D is the characteristic Debye frequency which is determined only by the material and the thickness of the device. In most cases for flexural mode resonators, the resonant frequency is much smaller than ω_D ($\omega_D \approx 3.7e7$ for the devices tested). In this case, the Q_{TED} is inversely proportional to the resonant frequency ω .

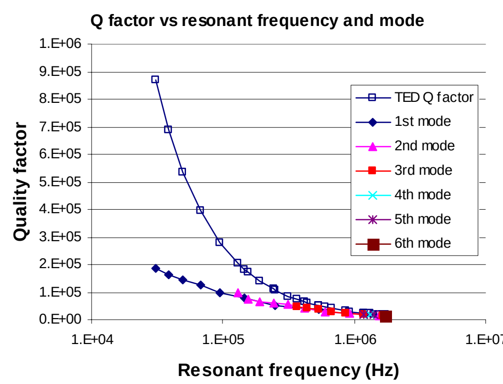

Figure 5 Comparison between TED and experimental results for different modes in vacuum

TRANSITION REGIME

The transition regime still holds our cloudiest understanding of damping mechanisms. Functionally this regime is useful for resonant pressure sensors. Generally there are two theoretical models to explain behavior in this regime: molecular damping and the "effective" viscous damping. For the molecular damping, it is believed that the molecular bombardment causes the damping [12]. For the "effective" viscous damping, it is believed that the viscous damping is still dominant with the "rarefied gas" effect [13].

By designing the experiment properly, we will be able to tell which damping mechanism is playing a dominant role. According to the molecular damping theory [12], the damping term β is independent of the width of the device, while for the viscous damping with “rarefied gas” effect [13], the damping term β depends on the width.

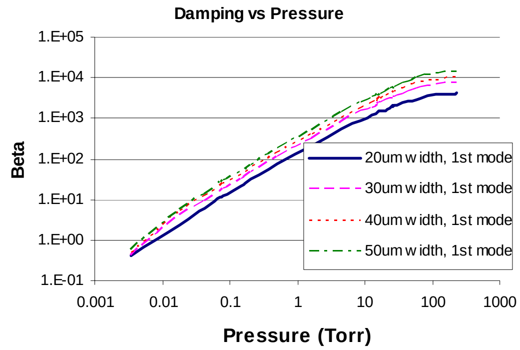

Figure 6 Pressure dependency o the damping for different widths under 1st mode resonation

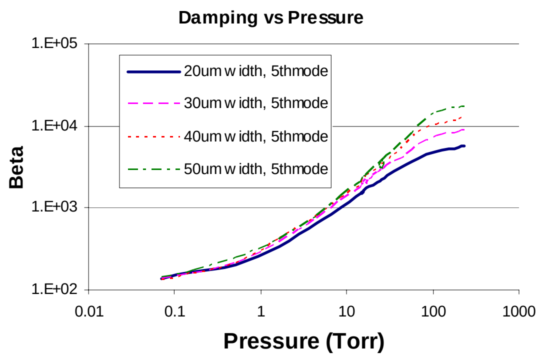

Figure 7 Pressure dependency o the damping for different widths under 5th mode resonation

The devices for this testing are shown in Figure 2 (5um gap). The length is 600um and the thickness is 5um. The widths are in range from 15um to 50um with 5um separation. The covered pressure range is from 3mTorr to the 760Torr. Up to 6 modes are tested. Results of 1st mode and 5th mode are shown in Figures 6-7. From Figure 6, the damping is pressure independent for pressure higher than 100Torr (squeeze film damping). Below that, the damping decreases with the decreasing of the pressure with the observation that the damping is dependent on the width. This strongly supports the “effective” damping theory. But the experimental results also imply that this width-dependence is becoming weaker when the pressure level goes lower. In addition, by comparing results of 1st mode (Rf~16 kHz) and 5th mode (Rf~900 kHz), we observe that for resonators with low resonant frequencies (or low resonant modes), the damping is sensitive to the pressure level even in low pressure range (3mT in our testing). This also explains the error we got between TED and the tested damping for low frequencies resonators in vacuum.

CONCLUSIONS

In this paper we systematically investigate the frequency dependence of the damping characteristics of MEMS cantilevers for 3 different pressure ranges: air pressure, vacuum and transition regime. In air pressure, a new frequency-dependent fluid damping model is proposed and validated experimentally. In vacuum, with the high Q we achieve, TED is shown to be the dominant damping source. In the transition pressure regime we experimentally support that viscous damping with “rarefied gas” effect dominates the molecular damping.

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