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# Using nonlinear dynamics for performance enhancement in resonant micro and nano-scale devices

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***Abstract* – MEMS and NEMS provide a novel and exciting medium where one can observe and utilize mathematical phenomena not often present in macro-scale systems. By understanding thoroughly the dynamics of resonant MEMS, we can not only predict device behavior to eliminate unwanted effects, but also use nonlinear effects to design better sensors and systems. This paper describes two examples of utilizing and exploiting the parametric resonance instability to design micro and nano-scale resonant sensors. The first example discusses a class of sensors (pressure and mass) based on tracking a sharp bifurcation present in the system. The second example, based on a coupled mode parametric resonance, is a parametric amplifier, which has direct applications in communications (filters, switches, mixers) where high-Q, tunable resonant elements are essential.**

## 1. INTRODUCTION

Resonant mode operation of microelectromechanical oscillators is common for many applications including on-chip mechanical filters/switches/mixers in the RF range, scanning probe microscopy [1] [2], small force detection, among other sensors. When operated in the linear regime as simple harmonic oscillators (SHO) there exists a tradeoff between the selectivity and the bandwidth of operation. In addition, in microscale components, the quality factor (selectivity) is primarily controlled by viscous damping which depends on the ambient conditions and cannot be controlled in open loop implementations. Parametrically resonant devices can be used to avoided a direct relationship between quality factor and selectivity/sensitivity.

Parametric resonance has been well established in many areas of science, including the stability of ships, the forced motion of a swing and Faraday surface wave patterns on water. We have previously investigated a linear parametrically driven torsional oscillator [3] and along with other groups have developed devices for applications including mass sensing[4] and parametric amplification [5]. Here, we present two examples of microscale applications which utilize parametric resonance.

## II. EXAMPLES

### *A. Parametric Resonance-based Mass Sensing*

One elegant application of parametric resonance in microscale devices is for highly-sensitive detection of mass change. This can be implemented as a chemical (vapor-phase) sensor. The device we use to carry out this study is an in-plane parametrically resonant oscillator. We show that in this configuration, the nonlinearities (electrostatic and mechanical) have a large impact on the dynamic response of the structure. This result is not unique at the microscale, as many MEMS oscillators display nonlinearities of equal importance (including the very common parallel plate actuator, and the electrostatically actuated cantilever beam resonator). Nonlinear effects can dramatically effect the behavior of microscale resonators. In the case of the mass sensor, a nonlinear Mathieu equation is used to model the device. Analytical results show that nonlinearity significantly changes the stability characteristics of parametric resonance [5]. Experimental frequency response around the first parametric resonance is well validated by theoretical analysis. Unlike parametric resonance in the linear case, the transitions (very critical for mass sensor application) from large response to zero happen at additional frequencies other than at the boundary of instability area.

The motivation to study and utilize nonlinear behavior in microscale sensors has been shown [4]. In the case of the mass sensor, we summarize some of the design issues surrounding the nonlinearities, both experimentally and analytically. The implications of these results on the behavior of the mass sensor are discussed.

The effect of the cubic mechanical stiffness and electrostatic stiffness changes the dynamic behavior of the oscillator response when excited parametrically in parts of the parameter (Frequency-Amplitude of input voltage) space. This restricts the regions of operation of the mass sensor, but does not alter the sensitivity. The detailed modeling and analysis also serve as tools for design of the mass sensor. The sharp transition, which facilitates the high mass change detection, can also be used to estimate the system parameters with good degree of accuracy in the reference oscillator.

For use as a mass sensor, additional issues need to be addressed. Understanding of the various sources of noises in the system is one main issue [6]. Also, we are investigating methods to activate the surface of the oscillator for selective reaction and perhaps increase the surface area using porous silicon. The effect of temperature on the frequency response of parametric resonance is another critical issue. These issues are currently being addressed.

### *B. Mode-coupled Parametric Amplifier*

Another application of parametric resonance phenomena in microscale systems is a parametric amplifier. While there exists previous work regarding parametric amplification in single microscale oscillators [7], this study describes parametric coupling between two distinct mechanical modes in a single oscillator. This two-mode non-degenerate parametric amplification offers some advantages over degenerate amplification. The most significant is that the driving frequency and response frequency are decoupled, as compared to degenerate parametric amplification, where they are related by integer or fractional harmonics. This

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allows for a method to decouple drive and sense (in a sensor) and reduce parasitic capacitive coupling.

## III. CONCLUSION

Using two examples in different application areas, we demonstrate the importance of nonlinear mechanics in microelectromechanical systems. The design of a mass sensor has been investigated in thorough detail, and the effect of nonlinearities, although significant, has been addressed. Secondly, a parametric amplifier which utilizes two interacting mechanical modes of a single oscillator. This self-resonance provides a simply manufactured, tunable parametric filter for applications in communication devices, and can be scaled to the RF range.

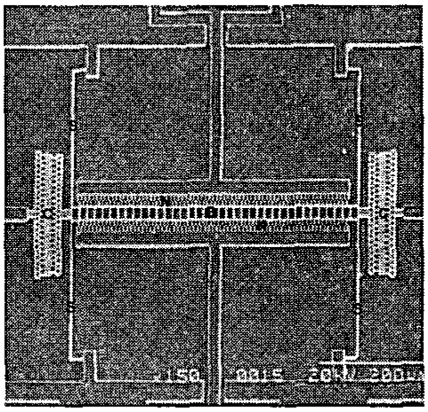

Figure 1. The device used to understand the coupled electrostatic and mechanical effects necessary for a parametrically resonant mass sensor. The device is electrostatically driven using the drives marked N. The drives marked C are for calibration only and are not used for device operation.

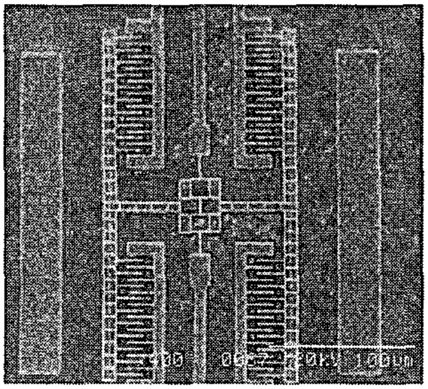

Fig.2 A microscale coupled-mode parametric amplifier. The device is made of Single Crystal Silicon (SCS) using the SCREAM-I process.

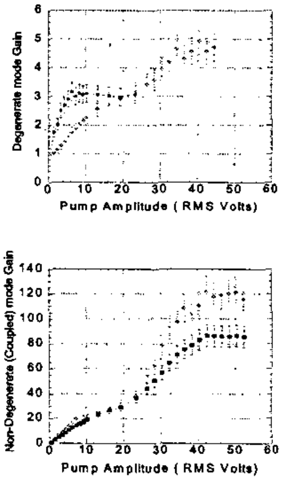

Figure 3. The MEMS parametric amplifier. The black squares are the first mode and the grey circles are the second mode of oscillation. The plot on the top shows the Gain (defined as the output velocity with **pump on / pump off**) of the two modes of oscillation as a function of the pump amplitude when pumped with a signal ~twice the corresponding natural frequency (degenerate .parametric amplification). The lower plot shows the respective gains of the two modes of response when pumped with the sum of the two modes (non-degenerate parametric amplification).At large pump signals, the gain saturates, as expected, due to self-oscillation and device non-linearity

## IV. ACKNOWLEDGMENTS

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